Global Warming as a System Response Theory Problem

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Abstract

Since the end of the "Little Ice Age" about 200 years ago, mean temperatures appear to have risen by about 1°C. To what extent is the rise due to increased energy input or increased "insulation"? The conventional approach is to construct detailed simulation models, try to fit the model parameters to historical data and infer trends in the global energy budget and their causes from the models. All steps in the process are laden heavily with theory and assumptions of very uncertain reliability. This paper proposes a complimentary approach which does not rely on estimating the global energy budget itself. The earth's climate is treated a "black box" system. System response theory is applied to the annual seasonal cycle in solar heating. As proof of principle, BADC Central England Daily Mean Temperature data from 1772 to 2005 were used to calculate the change in seasonal lag (a parameter not directly dependent on insolation intensity). The data show the seasonal lag has increased by about six days, presumably due an increase in the time-constant for cooling. At the same time, the summer-winter temperature difference initially decreased, but has been on a level trend since 1870. This latter behaviour is not consistent with an hypothesis of a temperature increase solely due to extra "insulation" by, say, CO_2. A great deal of climate physics is encoded in system response parameters like seasonal lag. Models should be tested against them.

Introduction

Since the end of the "Little Ice Age" about 200 years ago, mean temperatures appear to have risen by about 1°C. The most popular theory is that increase in atmospheric CO_2 since the industrial revolution, has increased the "insulation" of the earth. In this picture, Fourier's law dictates a temperature increase to restore a rate of heat-shedding into space that balances the incoming energy flux from the sun.

Elaborate computer models have been constructed, whose parameters are optimised on historical data. Those used by the IPCC predict that the current trend of increasing CO_2 concentration will cause global mean temperatures to rise several degrees over the course of the 21st century. However, it is hard to know whether the models have included all relevant effects and represented them correctly.

Some scientists believe the CO_2 warming effect is saturated, and that other factors (such as changes in insolation, and/or land use changes) can explain the warming observed to date.

This paper proposes a method with the potential to distinguish between external factors (such as insolation) from terrestrial factors (such as the posited CO_2 "insulation" effect) using temperature time series with sub-annual (preferably daily) time resolution.

Motivation

The proposal below is motivated by the technology of signal processing. It is very hard to detect a steady slow signal from noisy data with inputs subject to drift. If, however, one can modulate the signal sought with a high frequency "carrier", it becomes much easier to extract the signal from the noise.

System response theory proposal

Think of the earth climate system as a "black box" whose complex inner workings we don't understand. The earth is stimulated over its surface by a time-varying energy flux. We are able to measure the time-varying temperature response at certain probe points. What can we learn about this complex system by examining the response at particular frequencies? Consider

[1] $$T_\omega = J_\omega A_\omega \exp\left(-i\phi_\omega\right)$$

In equation [1], T_ω is the temperature response at frequency, ω, to solar stimulation J_ω at that frequency. The properties of the system leading to that response are all incorporated into the complex response function Fourier coefficient, $A_\omega \exp\left(-i\phi_\omega\right)$. Crucially, the phase lag, ϕ_ω, can be extracted from time series of temperature, T, without any need to know any more about J_ω than its phase.

The programme would be to perform Fourier analysis on the T time series in frames of, say, 4 to 8 years, and follow the trends of ϕ_ω and (ideally) A_ω. Even though one does not know J_ω, and thus A_ω is not really known, it may be possible to use a Kramers-Kronig[1] relation to derive it from ϕ_ω (up to a fixed constant). For purposes of calculating trends, the Fourier analysis frames could overlap.

The energy flux stimulation will have a strong component with a period of one-year and smaller components at higher harmonics. For example, **Figure** 1 shows a cosine fitted to the years 1772 to 1776. It illustrates well the difference between climate and weather: the cosine is the expected behaviour according to climate, while the weather is the experience on a particular day. Notice, for example, that at the time of the January 1773 cosine minimum, there was a warm spell with mean daily temperatures as high as 9°C.

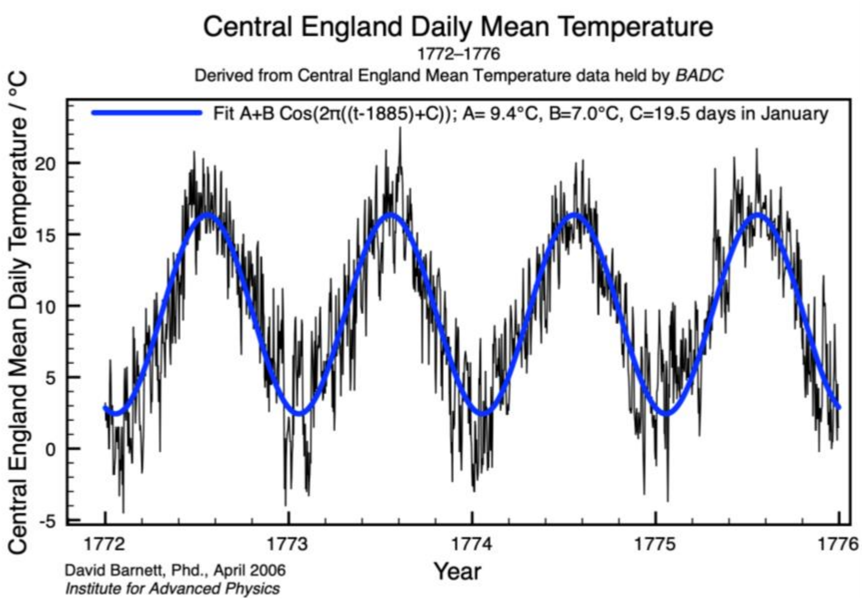

Figure 1: *Least Squares fit of cosine with a period of one year to daily mean temperature data in a block of four-years*

The most interesting parameter of the cosine fit is the seasonal lag, expressed as the date in January when the temperature is at a minimum. In the case of 1772 to 1776 that date was 19.5 days into January. Since the shortest day is around 20th December, that represents a lag of about 30 days which is close to $\pi/6$ radians.

To illustrate the kind of information that can be extracted, let us assume that the phase lag is

[1]. Reality and causality considerations mean that response function Fourier coefficients at different frequencies cannot vary arbitrarily and independently of one another. For example, the real and imaginary parts are related by a Hilbert Transform (known as a Kramers-Kronig relation in this context).

caused principally by storage of heat in the oceans[2]. We can represent the total storage effect by C_ω, and the "thermal resistivity" to the shedding of heat from the probe point into space by R_ω. The meaning of these symbols is summarised by **Figure** 2.

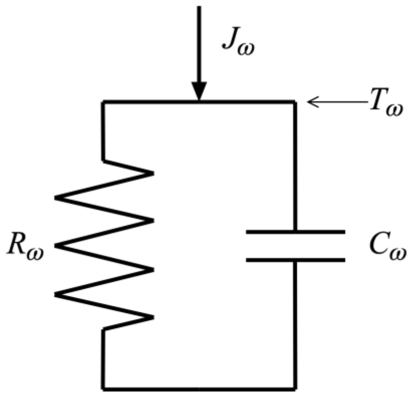

Figure 2: *The amplitude and phase of the response represented by analogy with an electrical circuit comprising a resistor and a capacitor. The analogy should not be taken literally, of course, because the "components" are valid only at the frequency of observation and encompass many effects including feedbacks.*

Conservation of energy requires that

[2] $$J_\omega = T_\omega \left(1/R_\omega + i\omega C_\omega\right).$$

The factor $i\omega$ arises from differentiation with respect to time.

Rearranging equation [2],

[3] $$T_\omega = J_\omega \frac{R_\omega}{\left(1+(\omega R_\omega C_\omega)^2\right)} \left(1 - i\omega R_\omega C_\omega\right)$$

Comparing with equation [1] we can identify

[4] $$A_\omega = \frac{R_\omega}{\left(1+(\omega R_\omega C_\omega)^2\right)^{1/2}}$$, and

[5] $$\exp\left(-i\phi_\omega\right) = \frac{1-i\omega R_\omega C_\omega}{\left(1+(\omega R_\omega C_\omega)^2\right)^{1/2}}.$$

Writing

[6] $$\psi_\omega = \tan\left(\phi_\omega\right) = \omega R_\omega C_\omega,$$

Equations [3], [4], and [5] become

[7] $$T_\omega = J_\omega \frac{R_\omega}{(1+{\psi_\omega}^2)} \left(1 - i\psi_\omega\right)$$

[8] $$A_\omega = \frac{R_\omega}{(1+{\psi_\omega}^2)^{1/2}}$$

[9] $$\exp\left(-i\phi_\omega\right) = \frac{1-i\psi_\omega}{(1+{\psi_\omega}^2)^{1/2}}.$$

The phase of J_ω

In what follows I shall be assuming that the northern and southern hemispheres of the earth are largely decoupled on the time-scale of a year, and that J_ω (with $\omega = 2\pi/(1 year)$) for the

[2]. **Figure** 2 does not represent explicitly the effect of winter snow and ice which increases the albedo and "short circuit"s some of the energy flow asymmetrically about the solstice. That could contribute to the temperature phase lag.

northern hemisphere is at a minimum on 20 December[3]. I would expect the higher harmonics to trough around the same time. This will allow us to read off the phase lag, ϕ_ω, from the temperature time series Fourier component with a period of one year.

Annual insolation cycle example and proof of principle using Central England Mean Daily Temperature Data from the BADC dataset

The stimulating energy flux has a strong Fourier component with period of one year. As a quick and dirty demonstration, the daily mean temperature time series for Central England was divided into blocks of 4 years duration. A cosine wave, with period one year, was fitted to each block (as in **Figure** 1, for example). The parameters for each block are Temperature Amplitude (i.e. half the summer to winter temperature change), T_ω, the temperature phase ϕ_ω (expressed as the date in January of minimum temperature), and the mean temperature for the block which we may as well call T_0. These three parameters have been plotted on a graph whose abscissa covers the years 1772 to 2005.

There are considerable fluctuations in the parameters from 4-year block to 4-year block. I have therefore provided an "eye-guide" to the trends in the form of cubic curves least-squares fitted to each parameter series. The results are summarised in **Figure** 3.

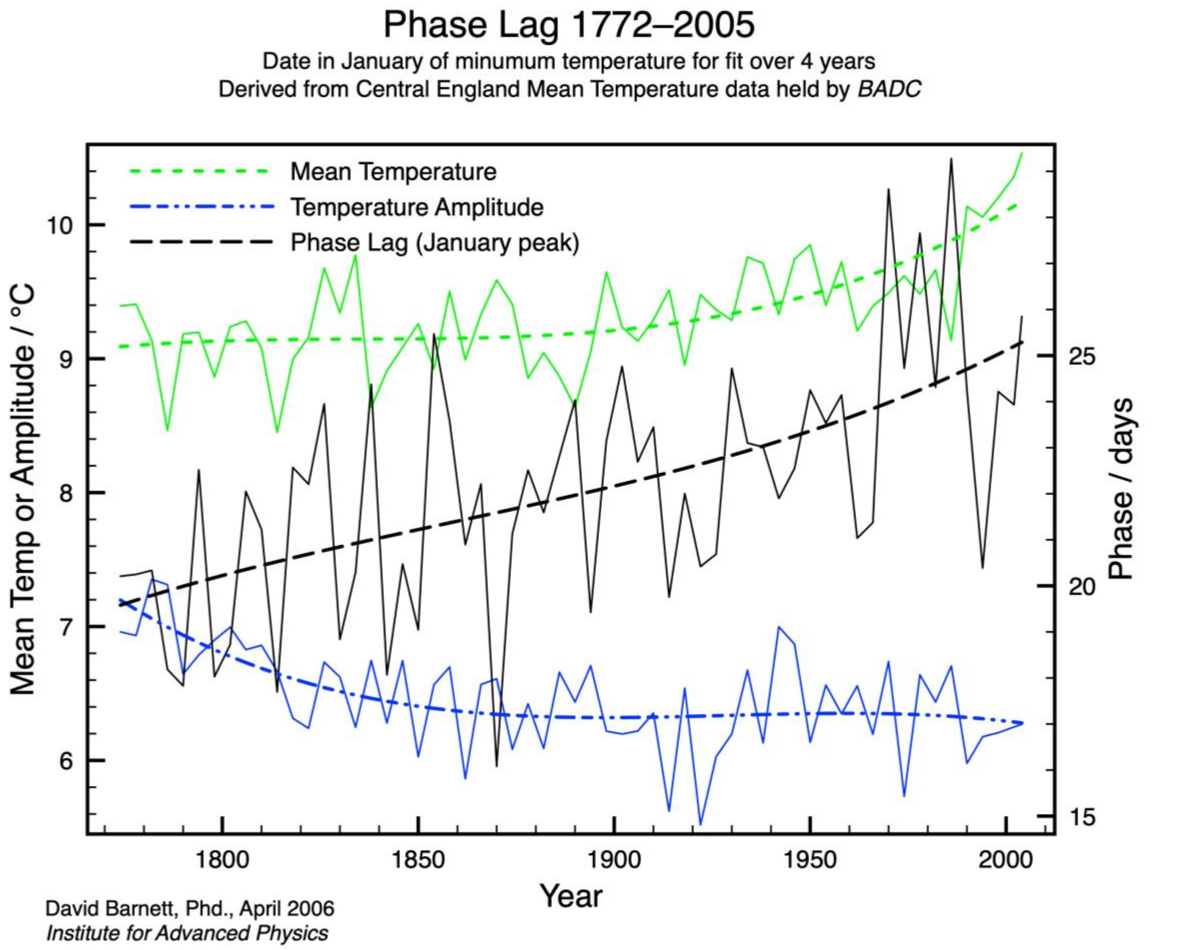

Figure 3: *Phase lag (date in January), temperature amplitude, and mean temperature of the 4-year blocks plotted for years 1772 to 2005, together with cubic trend lines fitted to the data.*

[3]. While this assumption for the phase of J_ω is plausible, it needs further investigation and justification.

Discussion of the trends in the Central England temperature series example.

First consider the seasonal phase lag. The lag can vary quite wildly in a short interval. Never-the-less there seems to be a secular trend for the coldest period, from around 20 January in 1800 to around 25 January in 2005.

Note that while I have been expressing the phase in terms of the coldest period, the fit to the data is going to be dominated by what happens in spring and autumn when the temperature is changing most rapidly from week to week. Thus while the phase fit implies a climate coldest around 20 January, there could easily be an "unseasonal" warm spell then (as happened in 1773—see **Figure** 1).

Depth of cold at 20 January 1800 is a 31 day lag (approximately $\pi/6$ radians) from 20 December 1799 (shortest day). 200 years later the lag seems to have increased to about 36 days (approximately $\pi/5$ radians) by 2004. That is an apparent increase lag trend seems to show an increase of about 5 days, (approximately $\pi/30$ radians) in 200 years.

If the data can be modelled by **Figure** 2, then the phase lag trend is consistent with a combination of an increasing "insulation" effect, R_ω, and the storage effect, C_ω.

Can the phase lag be attributed solely to an increase in R_ω?

Let us examine the hypothesis that the phase lag is due entirely to increasing insulation, and that the input flux amplitude, J_ω, remained constant.

Between 1772 and about 1870, temperature amplitude between summer and winter had a declining trend from about 7.2 °C to about 6.4 °C. It was more-or-less constant after 1870. Inspection of equation [8] shows that the hypothesis would predict increasing temperature amplitude. In fact, the decline in the early period cannot be matched assuming constant input flux, J_ω.

What is surprising is that matching the temperature amplitude history requires J_ω to *decline* between 1772 and 1870. For secular heating, it would be reasonable to expect J_ω also to increase. The discrepancy might come from the assumption that the minimum in solar flux converted to surface heat was fixed (and we assumed 20 December). However, as the climate warmed, the first snowfall would have become later. Light reflected from snow does not convert to surface heat. Later snow would mean delayed cooling.

If the spring thaw were also delayed (even remained unchanged), one could interpret the cooling delay as an effective delay in phase of J_ω (and hence ϕ_ω might be overestimated). However, since snowfall is a response to the seasonal cycle, it would be better to consider refining **Figure** 2 with other "circuit" elements (in some ways, the snowfall / spring thaw behaves mathematically like a component of C_ω).

If we ignore the considerations of the preceding two paragraphs, the contribution of changes in R_ω to the change in ϕ_ω is about 30%, while C_ω must contribute the rest. One can speculate that the increase in C_ω is due to an increase in the amount of ice-free sea during the summer.

Why constancy of summer-winter annual temperature change amplitude since 1870 implies that changes in R_ω contribute only around 30% to changes in seasonal lag.

Taking the logarithm of equation [8]:

[10] $$\ln A_\omega = \ln R_\omega - \tfrac{1}{2}\ln\left(1 + {\psi_\omega}^2\right)$$

and differentiating:

[11] $$\frac{\Delta A_\omega}{A_\omega} = \frac{\Delta R_\omega}{R_\omega} - \frac{{\psi_\omega}^2}{(1+{\psi_\omega}^2)}\frac{\Delta \psi_\omega}{\psi_\omega}$$

If $\Delta A_\omega = 0$ then

[12] $$\frac{\Delta R_\omega}{R_\omega} = \frac{{\psi_\omega}^2}{(1+{\psi_\omega}^2)}\frac{\Delta \psi_\omega}{\psi_\omega}$$

Noting that differentiating the log of the definition,[6], of ψ_ω gives

[13] $$\frac{\Delta \psi_\omega}{\psi_\omega} = \frac{\Delta R_\omega}{R_\omega} + \frac{\Delta C_\omega}{C_\omega}$$

From which it is evident that the fractional contribution of $\frac{\Delta R_\omega}{R_\omega}$ to $\frac{\Delta \psi_\omega}{\psi_\omega}$ is given by the coefficient of $\frac{\Delta \psi_\omega}{\psi_\omega}$ in equation [12].

Between 1870 and 2005, the seasonal lag increased from 33 days (0.57 radians) to 36 days (0.62 radians), corresponding to a change in ψ_ω from 0.64 to 0.71. Using an intermediate value to single digit precision gives

[14] $$\psi_\omega = 0.7 \Rightarrow \frac{{\psi_\omega}^2}{(1+{\psi_\omega}^2)} = 0.3$$

Inferring other global warming parameters and sanity check 1870–2005.

Following on from the discussion leading to equation [14], we note that

[15] $$\frac{\Delta \psi_\omega}{\psi_\omega} \approx 0.1 \Rightarrow \frac{\Delta R_\omega}{R_\omega} \approx 0.03$$

It is a fairly reasonable assumption that R_0 is close to R_ω. Under the assumption that J_0 has been constant over the period 1870 to 2005, we can infer from equation [2] that

[16] $$\frac{\Delta T_0}{T_0} = \frac{\Delta R_0}{R_0} \approx 0.03$$

Under the greenhouse warming hypothesis, T_0 should be the difference between the actual mean surface temperature (9°C) and the temperature that would prevail in the absence of all greenhouse gases (-18°C), which is about 27°C . That implies a total temperature increase, ΔT_0, of about 0.8°C, attributable to ΔR_0 in 135 years.

If we assume that ΔR_0 is due to entirely to added CO_2 and take the 1870 and 2005 atmospheric concentrations as 288 ppm and 380 ppm respectively, the partial doubling was $\log_2(380/288) = 0.4$. That implies a maximum sensitivity to CO_2 of 2°C per doubling.

Robustness of the system response theory approach to limited geographical data

As a teenager I was an electronics hobbyist. I would test an amplifier, say, by injecting a periodic signal at the input and then sample the response of the system at various points which did not necessarily have to be in the main signal path. Often one could detect signal, even on the DC supply rails (which is one reason why large capacitors across the supply were needed to prevent feedback oscillations). I mention this to highlight the fact that monitoring just a single point of a responsive system can provide useful information about the whole system.

Consider one of the pitfalls of the traditional approach which requires one to have good representation of the atmospheric temperature field over the whole globe and over extended periods of time. Yet the sampling is heavily biassed to densely populated areas, only a few places have time series going back more than a few decades. Even in those places that have extended time series, there is potential for inconsistencies between measurements taken decades apart (for example the growth of a stand of trees near the measuring station).

By contrast, a time series can yield seasonal phase data which is impervious to the inconsistencies in temperature base line across the decades. Further, even a single location may be representative a wide geographical area with varying terrain.

Developing the System Response approach

Apart from using time series from many more locations, one should do full Fourier transforms over time blocks varying from 1 year to, say, 32 years. One can use a phase-amplitude Kramers-Kronig relation to extrapolate and verify T_0 within a block of observations. The highest frequencies (beyond, say, monthly) are probably too noisy to be meaningful, but will not contribute much to the Hilbert Transform at the low frequencies of most interest (i.e annual to decadal). There is also a lot of scope for better analysis of the temperature time-series at each location.

Concluding remarks

This is proof of concept discussion, using a single location's dataset, with an unsophisticated analysis. The inferences, such as maximum possible temperature sensitivity to CO_2 are far from rigorous, but indicate some of the power of system response theory to tease out valuable climate physics from even limited observations.

Any serious attempt to model the climate should make use of as many features of the data as possible so as to reduce the danger of undetected hidden assumptions. The systems response approach, and seasonal lag in particular, is a promising tool in this regard. I note that some workers are beginning to use this approach in meteorology[4]. It does not seem too much of a leap to extend systems response theory to the global climate system.

[4]. Milan Palusˇ, Dagmar Novotna´ & Petr Tichavsky´, *GEOPHYSICAL RESEARCH LETTERS*, **32**, L12805 (2005) "*Shifts of seasons at the European mid-latitudes: Natural fluctuations correlated with the North Atlantic Oscillation*"