Viscous Dissipation Heating by Flows of Melted Ice on Greenland

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ABSTRACT

The contribution of viscous dissipation conversion of kinetic energy into thermal energy has been significantly over-estimated in three recent publications. The kinetic energy content of the macro-scale mean flow is assigned to be the heat dissipation into thermal energy. The estimate leads to temperature increases that make significant contributions to melting ice on Greenland.

A different estimate, in which the viscous dissipation is determined at the micro-scale of the flow, is calculated in these notes. This estimate, and the associated temperature increases in the flow, are significantly less than that based on the macro-scale.

I. INTRODUCTION

Three recent publications, Mankoff and Tulaczyk [2017], Karlsson *et al*. [2021], and Young *et al*. [2022], have provided estimates of the contribution of viscous dissipation to increases in the temperature of melt-water flows on the Greenland ice sheet. Note that the authors of the first publication are also authors for all three.

The estimate indicated that from 0.66 giga-Watt to 8.6 giga-Watt (GW) of energy was converted by viscous dissipation into thermal energy content of the flows. The associated increase in the temperature of the melt-water was calculated to be sufficient to cause additional significant melting of the Greenland ice.

Some initial rough estimates indicated that the power corresponded to the macro-scale energy content of the entire mean flow, and a thermal energy balance for the flow indicated that the dissipation would indeed significantly increase the fluid temperature.

The power estimate is too large by about at least an order of magnitude. Conversion by viscous dissipation into thermal energy occurs at the smallest spatial scales in the flow, the Kolmogorov scales, and not at the scale of the mean flow.

In these notes an estimate of the frictional losses associated with the flows is used to estimate small-scale viscous dissipation and its conversion into thermal energy. The estimate is based on an integrated averaged equation formulation for kinetic energy balance.

A video of an ice-melt lake draining down a hole was included with the Press Release [2022] Reference. The video is also available at: https://www.youtube.com/watch?v=vHJOIYEtNXA&t=61s. During the draining, which lasts a couple of hours, it appears that a mixture of air and water spurts upward at the top of the hole. That might be an indication that the flow does not completely fill the flow channel, and that a two-phase mixture is present down the hole.

II. EQUATIONS

Averaged formulations of the local-instantaneous Navier-Stokes equations for mass, momentum, and energy balance are frequently used in engineering analyses of physical phenomena and engineered equipment. Averaging generally reduces an intractable completely 3-dimansional problem to a tractable formulation, while at the same time include accounting of important aspects of the problem. The Bernoulli equation, an integral formulation of the Euler-Cauchy inviscid momentum balance is frequently used for this purpose. The Bernoulli equation, and other integral forms, are among the basic equations used in hydraulics and hydro-power; Vennard [1961] and Bower [1999], for examples.

Bird, Stewart, and Lightfoot, BSL hereafter, [1960, Chapter 7] develop and discuss the macroscopic balance equations for mass and momentum, and their relationship to the local-instantaneous partial differential equations. The idea is to integrate the 3-dimensional equations over large chunks of equipment, that may contain other equipment such as pumps and compressors, to get equations that describe the flows at the macroscopic scale of the mean flow. In contrast to the Bernoulli equation, these equations include accounting of viscous and thermal effects in the flows.

The developments in BSL are a summary of the detailed derivation given by Bird [1957], and discussions by Synolakis and Badeer [1989]. The derivation given by Bird is summarized by BSL in Example 7.3-1. Additional in-depth details have been investigated by Wang *et al.* [2021].

The primary interest for these notes is the macroscopic generalized mechanical energy equation. Within that equation the focus is in the viscous dissipation of kinetic energy into thermal energy. The derivation by Bird

[1957, Equation 20] begins with the local-instantaneous form of the kinetic energy equation because the interest is in conversion of kinetic energy into thermal energy.

For application to the flows on Greenland the following assumptions are made. The flow is steady state, constant density water, the channel flows full with water, no work is performed by the water on the wall, the averages of products can be replaced by products of averages, and the channel is roughly circular.

Regarding the assumption of a full channel, as the lake drains and the fluid available for discharge decreases, it is likely that the channel is not full. A video in the News Release show spouting/spurting of a water-air mixture up out of the hole as the lake drains. Two-phase flows will affect the estimate of the frictional losses used in these notes; Saito *et al.* [1978], however, details needed for more nearly complete modeling are not available, and very likely will not be.

For these conditions, Equation 7.3-2 in BSL, Eq. 29 in Bird, gives

$$\Delta\left[\frac{1}{2}V^2 + \phi + \frac{P}{\rho}\right]_1^2 + \hat{E}_v = 0 \tag{1}$$

where V is the flow velocity, ϕ is the potential energy, P is pressure, ρ is water density, and Δ means that integration along the flow path has been varied out.. The kinetic energy representation, the first term, has dropped the turbulent fluctuations as these are generally considered small.

The last term in Eq. (1) $\hat{E}_v$ is the “friction loss”, the rate of irreversible conversion of mechanical energy into thermal energy. The terms are per unit mass flow rate so that $\hat{E}_v = E_v / W$, $W = Q\rho = \rho V A_f$. The dissipation term, E_v is the integration of the local-instantaneous dissipation over the volume of the flow. See BSL and Bird references for details. An application of modeling dissipation for engineering flows has been investigated by Hughes and Duffey [1991]. The book by Arpaci [2019] illustrates the utility of micro-scale modeling for a variety of engineering applications involving turbulent flows.

Equation (1) says that the state at the end of the flow path is equal to the state at the initiation minus the viscous dissipation

$$\left[\frac{1}{2}V^2 + \Phi + \frac{P}{\rho}\right]_2 = \left[\frac{1}{2}V^2 + \Phi + \frac{P}{\rho}\right]_1 - \hat{E}_v = 0 \qquad (2)$$

BSL beginning at Section 7.4 develop expressions for the dissipation for a few flow situations. For flows in a simple closed channel like a straight pipe which will be used in these notes, the dissipation is [equation 7.4-9 in BSL]

$$\hat{E}_v = \frac{1}{2}V^2 \frac{L_f}{R_{hy}} f_f \qquad (3)$$

where L_f, R_{hy} and f_f are the length of the flow path, the hydraulic radius of the channel, and friction factor, respectively.

III. THE DATA

Young *et al*. and the SI [2022] give some data that was used in calculation of the heat dissipation. The values that needed for the present calculation are summarized here.

The details of the cross-sectional flow area and wall roughness of the flow channel from top to bottom are not known. The hydraulic radius of the channel, $R_{hy} = A_f / P_w$ is reported to be 2 m. Reynolds number and friction factor calculations require the hydraulic diameter, the equivalent wetted diameter, $D_{hy} = 4A_f / P_w$. For the assumed circular flow channel these give the diameter and D_{hy} to be 8 m. The diameter will be used to convert reported volumetric flow rates to flow speeds and mass flow rate, $w_w = \rho_w V A_f$, down the channel. The depth of the channel is reported to be 605.6 m. The flow area is calculated using the conversion of hydraulic radius to diameter given just above, and based on that area the velocity is also calculated given the volumetric flow rate.

The volumetric flow rates are obtained by converting the depth of melting per day to volume per day. The maximum melting of 57 mm d^{-1} corresponds to

$80x10^6$ m^3 d^{-1}, minimum melting is reported to be 4 mm d^{-1} which converts to $5.75x10^6$ m^3 d^{-1}. The latter value is reported in the Surface Driver of Basal Melting section. The Basal Heat Transfer section reports also 10 mm d^{-1}. The average volumetric flow rate is given as $16x10^6$ m^3 d^{-1}. The viscosity of water is taken to be $1.8x10^{-6}$ m^2 s^{-1}.

As neither the micro- or macro-scale details of the channel walls are known, getting a value for the friction factor is somewhat fuzzy. Calculations using a standard engineering equation for high Reynolds Number and smooth walls gives low friction factor values, as it should for these large Reynolds Number flows. It order to provide an upper bound for the viscous dissipation conversion to thermal energy, the value for micro-scale fully-rough walls will be used in Eq. (3), $f_f = 0.05$. Significant macro-scale structure of the channel walls, appearing somewhat like local flow-path distortions and thus local energy losses, would increase the viscous dissipation. A general form accounting for wall and local losses is given by BSL

$$\hat{E}_v = \sum_i \left(\frac{1}{2} V^2 \frac{L_f}{R_{hy}} f_f \right)_i + \sum_i \left(\frac{1}{2} V^2 e_v \right)_i \qquad (4)$$

where e_{vi} is the loss factor for local flow perturbations, the first sum is over the straight sections of the channel, and the second for each local perturbation. If the structure of the channel was known, variations in the structure could be taken into account and Eq. (4) might improve estimates of the frictional losses.

IV. MICRO-SCALE IRREVERSIBLE DISSIPATION

Using the above information, the viscous dissipation conversion to thermal energy for the season-wide-average melt volumetric flow of $16x10^6$ m^3 d^{-1}, the viscous dissipation is about 0.0190272 Gw. A small amount of energy that would increase the temperature of the mean flow by very little. Viscous dissipation goes linearly with the friction factor. If the real-world friction loss is less than the value given above, viscous dissipation will be less, and *vice versa*.

V. HEAT DISSIPATION IN THE PAPERS

The analyses in Mankoff and Tulaczyk [2017] and Young *et al*. [2022] are based on the macro-scale mean flows. They report, in the Surface Driver of Basal Melting section, that 4 mm d^{-1} melting produces 0.66 GW of power and 57 mm d^{-1} produces 8.6 GW. The latter melting seems to correspond to a volumetric flow of $80x10^6$ m^3 d^{-1}.

For the season-wide-average melting volumetric flow of $16x10^6$ m^3 d^{-1}, the complete potential energy content of the mean flow is about 1.1 Gw. That is essentially the value that the paper assigns to the viscous dissipation.

After starting to work on these notes, I found the paper by Karlsson *et al*. [2021]. Following equation (8) in that paper the authors explicitly state that it is assumed that the entire potential energy content of the mean flow is converted by viscous dissipation into thermal energy. An assumption that is not in agreement with the engineering literature.

Viscous dissipation estimated above is about two orders of magnitude less that the potential energy content of the mean flow.

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